BIN PACKING: SET COVERING FORMULATION

$\mathcal{S} = \{$ family of (maximal) packings of a bin $\}$

$$x_S = \begin{cases} 1 & \text{if } \cancel{\ } \text{ set } S \text{ is assigned to a bin} \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{S \in \mathcal{S}} x_S \qquad\qquad \min \sum_{S \in \mathcal{S}} x_S$$

$$\sum_{S \in \mathcal{S} : i \in S} x_S \geq 1 \quad i = 1, \dots, m \qquad \Longleftrightarrow \qquad \sum_{S \in \mathcal{S}} a_S^i x_S \geq 1 \quad i = 1, \dots, m$$

$$x_S \in \{0,1\} \quad S \in \mathcal{S} \qquad\qquad x_S \in \{0,1\} \quad S \in \mathcal{S}$$

$$a_S^i = \begin{cases} 1 & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases}$$

COLUMN GENERATION:

$\exists\, S^* \in \mathcal{S} : \sum_{i \in S^*} \pi_i^* > 1$ equivalent $\sum_{i=1}^{m} a_{S^*}^i \pi_i^* > 1$

KP01

$$\max \sum_{i=1}^{m} \pi_i^* z_i$$

$$\sum_{i=1}^{m} w_i z_i \leq W$$

$$z_i \in \{0,1\} \quad i = 1, \dots, m$$

$$S^* = \{ i : z_i = 1 \} \qquad \Longleftrightarrow \qquad a_{S^*}^i = z_i$$

Cutting Stock Problem

- m <u>item types</u> $i = 1, \dots, m$ weight w_i demand d_i

to be packed in the ~~minimum~~ min number of identical bins of capacity W. (This is like a BPP with multiple copies of the items).

Feasible Packing: encoded as a m-component vector $a_s = [a_s^1, \dots, a_s^m]$ where a_s^i: # of copies of item i

<u>ILP model</u>

$$\min \sum_{s \in S} x_s$$

$$\sum_{s \in S} a_s^i x_s \geq d_i \quad i = 1, \dots, m \qquad \left(\begin{array}{c}\text{DUAL VAR} \\ \pi_i\end{array}\right)$$

$$x_s \geq 0 \text{ INTEGER}, \quad s \in S$$

x_s = # of times packing s is assigned to a Bin

S = family of feasible packings,

i.e., $s \in S \iff \sum_{i=1}^{m} a_s^i w_i \leq W$, $a_s^i \leq d_i$
(maximal)

S = family of <u>maximal</u> packings

$$s \in S \iff \sum_{i=1}^{m} a_s^i w_i \leq W \ , \ a_s^i \leq d_i \ ,$$

$$\forall j : a_s^j < d_j \ , \ \sum_{i=1}^{m} a_s^i w_i + w_j > W$$

COLUMN GENERATION : given x^*, π^*,

$\exists s^* : \sum_{i=1}^{m} a_{s^*}^i \pi_i^* > 1$?

BOUNDED KNAPSACK PROBLEM

$$\max \sum_{i=1}^{m} \pi_i^* z_i$$

$$\sum_{i=1}^{m} w_i z_i \leq W$$

$$0 \leq z_i \leq d_i \text{ INTEGER } i = 1, \dots, m$$

where z_i = # of copies of item i in s^*,

i.e., $a_{s^*}^i = z_i$